Suppression of noise in nonlinear systems subjected to strong periodic driving

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An overdamped Brownian motion in "quartic" potential subjected to periodic driving has been considered. This system for the case of a weak periodic driving has been intensively studied during past decade within the context of stochastic resonance. It has been demonstrated that for the case of predominantly suprathreshold driving (the driving amplitude is significantly larger than the static threshold) the noise in the output signal is strongly suppressed at a certain frequency range: the signal-to-noise ratio demonstrates resonant behavior as a function of frequency.

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The switching dynamics of an overdamped Brownian particle in a "quartic" potential has been intensively studied in the past decade in the frame of stochastic resonance (SR) phenomenon [1]. Stochastic resonance is a nonlinear noise-mediated cooperative phenomenon wherein the coherent response to a deterministic signal can be enhanced in the presence of an optimal amount of noise. It had been observed in a wide variety of electronic systems, such as, lasers [2], Schmitt triggers [3], tunnel diodes [4], and superconducting quantum interference devices (SQUIDs) [5,6]. It is known that for a single-level threshold system, SR has been observed for the case of weak (underthreshold) driving [1] (although residual SR effects are known to occur for marginally suprathreshold signals [7]). In this case, as well as in the case of suprathreshold SR in multilevel threshold systems [8], the manifestation of SR is the resonant behavior of the signal-to-noise ratio (SNR) or other relevant characteristics as a function of noise intensity.

However, most of practical devices are operating in the predominantly suprathreshold regime, when the transition from one state to another one over a potential barrier occurs deterministically and noise is the only disturbing factor leading to erroneous switching. As an example of such a situation we can refer to the microwave hysteretic SQUID [9,10], whose dynamics is described by the model of Brownian motion in a bistable potential subjected to strong periodic driving at the given frequency. Such a SQUID represents a clear example of the device, by its very basic idea operating in strongly suprathreshold regime (its important characteristic is a function of the amplitude of the output signal versus the amplitude of the input driving) and for which the presence of noise leads to earlier transitions over the potential barrier that results in an error of the measured dc magnetic flux. For microwave hysteretic SQUID a long-standing problem is known: that of determining the parameters at which it should operate in order to demonstrate maximal sensitivity. On one hand it is known that with the increase of pumping frequency the sensitivity of the SQUID should improve, on the other hand it is known that at frequencies higher than the cutoff frequency, the performance of the device should degrade. Some qualitative treatment of this problem has been done in the past [9,11], but the question is still open.

Few papers, where the case of strong periodic driving was considered both in classical [12–14] and quantum systems [15], were addressed to investigate the area of hysteretic loop and its resonant behavior as a function of the frequency of driving signal was demonstrated. But, to the best of our knowledge, the investigation of the signal-to-noise ratio as a function of frequency was not performed for the case of strongly suprathreshold driving. This may be explained by the fact that most of the studies were restricted by adiabatic approximation, where frequency dependence of SNR could not be investigated in detail, or by linear response theory, where the driving amplitude was assumed to be small. Nevertheless, in the frame of linear response theory some weak resonant frequency dependence of the signal-to-noise ratio was recently observed for a particular case of piecewise rectangular potential [16].

Recently we have shown, using description via temporal characteristics, that in a dynamical system with noise subjected to a strong periodic driving, significant suppression of noise is possible in a certain frequency range [17]. The manifestation of such suppression is the very weak dependence of the mean transition time on the noise intensity in a certain frequency range.

In the present paper we consider fluctuational dynamics of a Brownian particle in a "quartic" potential subjected to strong periodic driving and study the signal-to-noise ratio at the given driving amplitude as a function of frequency and noise intensity. If in the low frequency case we start from the predominantly suprathreshold driving, with approaching the cutoff frequency the dynamic threshold amplitude being a function of frequency will reach the driving amplitude and above the cutoff frequency the particle will not be able to escape from one state to another in the absence of noise. In this case we observe resonant behavior of SNR as a function of frequency and the maximum is located near the cutoff frequency that approximately corresponds also to the time-matching condition (the mean transition time from one state to another will be minimal). In the case where the driving frequency is higher than the cutoff frequency we observe resonant behavior of SNR as a function of noise intensity. While the latter effect has already been studied in [7], to the

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best of our knowledge the resonant behavior of SNR as a function of frequency is, for the first time, observed for the case of strong driving and the effect may be used for suppression of noise in real electronic devices.

Consider a process of Brownian diffusion in a potential profile

$$U(x,t)=bx^4-ax^2+xA\sin(\Omega t+\varphi), \tag{1}$$

where φ is initial phase. It is known that the probability density $W(x,t)$ of the Brownian particle in the overdamped limit (Markov process) satisfies the Fokker-Planck equation

$$\frac{\partial W(x,t)}{\partial t}=-\frac{\partial G(x,t)}{\partial x}=\frac{1}{B}\left\{\frac{\partial}{\partial x}\left[\frac{du(x,t)}{dx}W(x,t)\right]+\frac{\partial^2 W(x,t)}{\partial x^2}\right\}. \tag{2}$$

Here $G(x,t)$ is the probability current, $B=h/kT$, h is the viscosity (in computer simulations we put $h=1$), T is the temperature, k is the Boltzmann constant, and $u(x,t)=U(x,t)/kT$ is the dimensionless potential profile. The initial and the boundary conditions have the following form:

$$W(x,0)=\delta(x-x_0), \quad G(\pm\infty,t)=0. \tag{3}$$

In computer simulations we chose the following parameters of the potential: $b=1$, $a=2$. With such a choice the coordinates of minima equal $x_{\min}=\pm 1$, the barrier height $\Delta U=1$, the critical (threshold) amplitude A_c at $\Omega\to 0$ is around 1.5, and we have chosen $A=2$ to be far enough from A_c. We note that this is not the case "just above the threshold level," considered in [7], but indeed strong driving: the amplitude $A=2$ is far above the dynamic threshold in rather broad frequency range. We also performed the analysis for $A=3,4,5$: the results are qualitatively the same, only *SNR* rises accordingly and the location of maximum of *SNR* is shifted to higher frequencies.

The quantity of our interest is the *SNR*. In accordance with [1] we denote SNR as

$$\mathrm{SNR}=\frac{1}{S_N(\Omega)}\lim_{\Delta\omega\to 0}\int_{\Omega-\Delta\omega}^{\Omega+\Delta\omega}S(\omega)d\omega, \tag{4}$$

where

$$S(\omega)=\int_{-\infty}^{+\infty}e^{-i\omega\tau}K[\tau]d\tau \tag{5}$$

is the spectral density, $S_N(\Omega)$ is noisy pedestal at the driving frequency Ω, and $K[\tau]$ is the correlation function

$$K[\tau]=\langle\langle x(t+\tau)x(t)\rangle\rangle, \tag{6}$$

where the inner brackets denote the ensemble average and outer brackets indicate the average over initial phase φ.

In order to obtain the correlation function $K[\tau]$ we solved Eq. (2) numerically, using the Crank-Nicholson scheme.

In Fig. 1 the spectral density $S(\omega)$ is presented for $kT=0.1$ (the delta spikes at the first and some higher harmonics are outside of the figure in order to enhance the noise part). One can see that the form of $S(\omega)$ significantly depends on driving frequency Ω, while the amplitude of the output signal is a monotonically decreasing function of Ω. In order to study the resonant behavior of spectral density, let us plot the SNR as a function of driving frequency Ω. From Fig. 2 one can see that SNR as a function of Ω has a strongly pronounced maximum. The location of this maximum at $\Omega=\Omega_{\max}$ is close to the cutoff frequency and approximately corresponds to the time scale matching condition: $\Omega_{\max}\approx\pi/\tau_{\min}$, where $\tau_{\min}$ is the minimal transition time from one state to another (see the definition of transition time as well as its investigation as a function of noise intensity and driving frequency in Ref. [17]). The existence of optimal driving frequency may be explained in the following way. Let us consider the case of adiabatically slow driving. If noise is

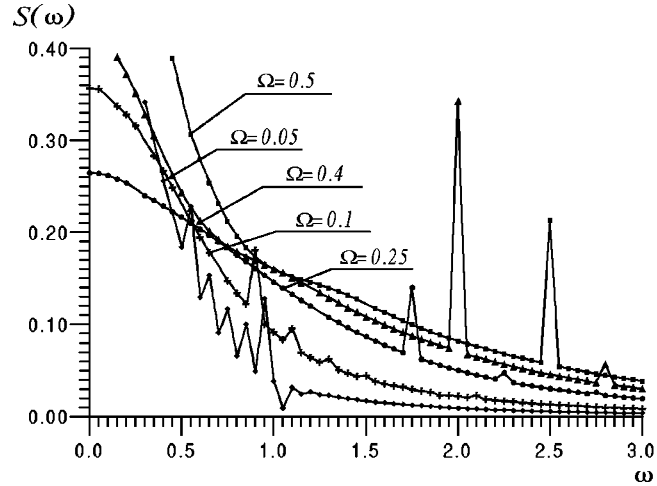

FIG. 1. Spectral density $S(\omega)$ with enhanced noise part, $kT=0.1$, $A=2$; dots are connected by solid lines as a guide to the eye (dimensionless units).

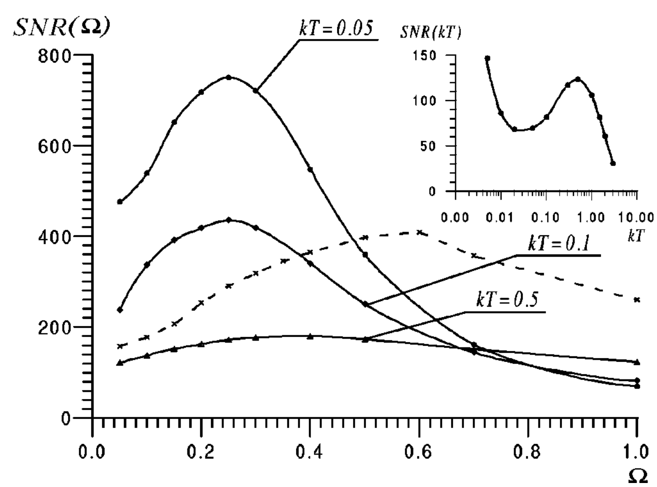

FIG. 2. Signal-to-noise ratio as a function of driving frequency for $A=2$ (dimensionless units); by crosses and dashed line, SNR for $A=3$ and $kT=0.1$ is presented, the curve is reduced by the factor of 5. Inset: SNR as function of kT for $\Omega=1$, $A=2$.

absent, the escape would occur only after the corresponding potential barrier would disappear. If we add some small amount of noise, the escape would occur earlier than in the deterministic case at some nonzero barrier height (we consider escape in a probabilistic sense, say as decay of probability *e* times). If the driving frequency is increased, the potential barrier height will decrease faster and the escape will occur at a lower barrier, that is, closer to the deterministic case (see the investigation of probability evolution in time-periodic potential in [17]). In the case where the driving frequency is higher than the cutoff frequency of the system, $\Omega \geqslant \Omega_c$ (where Ω_c has a dynamical sense: for a given amplitude of the signal the performance of the system significantly degrades above a certain frequency), the particle would never escape over the potential barrier in the absence of noise and will remain in the vicinity of the initial potential minimum, since there is not enough time to reach the basin of attraction of another state. Therefore, there is some frequency range, where at the given small noise intensity the escape will occur over the smallest potential barrier and in this case noise has a minimal effect on the system that results in the maximal SNR.

In the case where the driving frequency is higher than the cutoff frequency, $\Omega \geqslant \Omega_c$ (the driving amplitude is below the dynamic threshold), adding some amount of noise will help the particle to move to another state and the conventional stochastic resonance [7] may be observed (see the inset of Fig. 2 for $\Omega = 1$). We note that even in the case $\Omega \geqslant \Omega_c$, the SNR infinitely rises for $kT \rightarrow 0$, since, for $kT = 0$, there will be small but nonzero oscillations near the initial potential minimum.

In conclusion we have shown that in the dynamical system with the noise driven by a strong sinusoidal signal (predominantly suprathreshold driving), the influence of noise is significantly reduced in a certain frequency range: the signal-to-noise ratio is a resonant function of the frequency of the driving signal. This effect is of real importance for applications since it may allow one to operate a concrete device (like a SQUID or other electronic devices) in the regime of minimal noise-induced error.

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